# 5d-4f Emission of Nd<sup>3+</sup>, Sm<sup>3+</sup>, Ho<sup>3+</sup>, Er<sup>3+</sup>, Tm<sup>3+</sup> Ions in Alkaline Earth Fluorides

E. Radzhabov, V. Nagirnyi, M. Kirm, and E. Prosekina

Abstract—Emission and excitation spectra of CaF<sub>2</sub>, SrF<sub>2</sub>, BaF<sub>2</sub> doped with Nd<sup>3+</sup>, Sm<sup>3+</sup>, Ho<sup>3+</sup>, Er<sup>3+</sup>, Tm<sup>3+</sup> excited by vacuum ultraviolet photons or by x-rays were studied. Decay times of the Nd<sup>3+</sup> emission near 180 nm at 7 K were 17.7, 15.3, 12.3 ns in CaF<sub>2</sub>, SrF<sub>2</sub>, BaF<sub>2</sub> respectively.  $4f^{n-1}5d^{1}-4f^n$  luminescence bands were found at 170–200 nm for Sm<sup>3+</sup> ions and at 140–190 nm for Ho<sup>3+</sup>.

No excitation of 5d-4f emission of  $Nd^{3+}$  and  $Tm^{3+}$  was observed in the region of band-to-band transitions below 127 nm (BaF<sub>2</sub>), 120 nm (SrF<sub>2</sub>) and 115 nm (CaF<sub>2</sub>).

Emission spectra of Nd and Er doped alkaline-earth fluoride crystals under x-ray excitation were studied with concentration of impurity from 0.01 to 10 molar %. For the Nd<sup>3+</sup> and Er<sup>3+</sup> 5d-4f emissions in SrF<sub>2</sub> the light yields were estimated as 30–40 ph/Mev.

*Index Terms*—Energy transfer, excitons, scintillator, vacuum ultraviolet.

## I. INTRODUCTION

**MISSION** from the 5d to 4f shell of rare earth ions doped into ionic crystals was extensively studied in the last decade [1]–[3]. The emission observed in vacuum ultraviolet region has potentiality for exploitation in lasers, scintillators or quantum cutting systems.

From the general point of view the decrease of emission wavelength should lead to the decrease of decay time [4]. Due to this, many attempts were made to find dense and effective scintillation material doped with rare earth ions having emission in vacuum ultraviolet [5]. Among all 14 lanthanide ions only half, the ions with the number of f-electrons up to seven, can be considered as promising for fast scintillating. The heavier lanthanide ions posses slow vacuum ultraviolet emission because the lowest excited  $4f^{n-1}5d^1$  state has a larger spin (usually designated as high spin-HS) than the ground  $4f^n$  state, so the  $4f^{n-1}5d^1$ - $4f^n$  transitions become forbidden by spin rule.

Light lanthanides  $Ce^{3+}$ ,  $Pr^{3+}$ ,  $Nd^{3+}$  show bright  $4f^{n-1}5d^{1}-4f^{n}$  luminescence under optical excitation even

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E. Radzhabov and E. Prosekina are with the Vinogradov Institute of Geochemistry, Russian Academy of Sciences, 664033 Irkutsk, Russia (e-mail: eradzh@igc.irk.ru).

V. Nagirnyi and M. Kirm are with the Institute of Physics, University of Tartu, 51014 Tartu, Estonia.

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at room temperatures, while  $Gd^{3+}$ ,  $Lu^{3+}$  have luminescence at low temperature only [6]. Among the heavy ions only  $Er^{3+}$ ,  $Tm^{3+}$  [7],  $Ho^{3+}$  [8], [9] and  $Tb^{3+}$  [10] show  $4f^{n-1}5d^{1}-4f^{n}$ vacuum ultraviolet luminescence. The emission peaks were assigned to the spin-allowed (low spin LS to low spin) and the spin-forbidden (high spin-HS to low spin) transitions between the levels of the  $4f^{n-1}5d$  and the levels of the  $4f^{n}$  electronic configuration of the heavy lanthanide ion [2]. Recently we found a weak vacuum ultraviolet luminescence of Ho in CaF<sub>2</sub> [12]. Additionally to this we found now a bright Ho<sup>3+</sup> luminescence in SrF<sub>2</sub> and BaF<sub>2</sub> and a weak Sm<sup>3+</sup> vacuum ultraviolet luminescence in all three alkaline-earth fluoride hosts. It should be noted that the Sm<sup>3+</sup> vacuum ultraviolet luminescence was not yet known in any compound.

It is generally known that the light output of rare-earth doped scintillators decreases in the row of impurities  $Ce^{3+}$ ,  $Pr^{3+}$ ,  $Nd^{3+}$  [11], however the efficiency and mechanisms of the electron-hole or exciton energy transfer in such scintillators are yet poorly investigated.

The aim of the present paper is to investigate the ultraviolet 5d-4f emission of rare-earth ions and mechanisms of energy transfer in alkaline-earth fluoride crystals  $CaF_2$ ,  $SrF_2$ ,  $BaF_2$  by vacuum ultraviolet spectroscopy. Some previous results were published in a paper [12].

### II. EXPERIMENTAL

Crystals CaF<sub>2</sub>, SrF<sub>2</sub>, BaF<sub>2</sub> doped with REF<sub>3</sub> (RE-Nd, Sm, Eu, Gd, Tb, Ho, Er, Tm, Yb) were grown by the Stockbarger method in a graphite crucible in vacuum. Normally six or more rods with concentration of dopant from 0.005 to 10 molar percent were grown. At concentration less 0.3 mol.% the samples were cutted from grown rod. At higher concentration the sample were sawed from the rod and polished.

Emission spectra in vacuum ultraviolet region were measured with a solar blind photomultiplier FEU142 attached to a VM4 grating vacuum monochromator. Samples were excited either by Xe discharge lamp emitting at 147 or by x-ray tube. The excitation and absorption spectra in the energy range of 4-13 eV were measured with a grating vacuum monochromator VMR2. A discharge deuterium lamp L7293 with a MgF<sub>2</sub> window (Hamamatsu) was used as a vacuum ultraviolet source. Absorption in a vacuum ultraviolet region was measured for all samples at room temperatures to check the absence of unwanted impurities.

Time-resolved spectra were recorded using synchrotron radiation at the SUPERLUMI station of HASYLAB at DESY (Hamburg, Germany) [13].



Fig. 1. Excitation spectra of  $Nd^{3+}$  emission in  $CaF_2$ ,  $SrF_2$ ,  $BaF_2$  crystals. Inset shows the decay curves of the  $Nd^{3+}$  emission.

For simplicity we used a simpler f-d or 4f–5d notation instead of full notation  $4f^{n}-4f^{n-1}5d^{1}$ .

## **III. RESULTS**

# A. $Nd^{3+}$ Emission

The emission spectra of the  $Nd^{3+}$  ions in vacuum ultraviolet in all three alkaline-earth fluorides were measured in our previous paper [12]. The details of emission were satisfactory explained by the transitions from the lowest 5d level to different  $4f^n$  terms of the ground state.

The decay times of the Nd emission under optical excitation in the region 180–190 nm at 7 K were 17.7, 15.3, and 12.3 ns in CaF<sub>2</sub>, SrF<sub>2</sub>, and BaF<sub>2</sub>, respectively. The Nd emission decay curves were single exponential when excited in the lowest absorption band 170–180 nm. The rise times of 3–4 ns



Fig. 2. Emission spectra of Sm<sup>3+</sup> in CaF<sub>2</sub>, SrF<sub>2</sub>, BaF<sub>2</sub> crystals.

were observed under illumination into the higher lying excitation bands in  $CaF_2$  and  $SrF_2$ . No any rise of emission was observed in  $BaF_2$ -Nd. It seems that a smaller splitting of the 5d levels ( $e_g$ - $t_{2g}$ ) in  $BaF_2$  increases significantly the probability of non-radiative transitions to the lowest 5d excited level.

The vacuum ultraviolet 5d-4f emission of Nd<sup>3+</sup> was excited only in the crystal transparency region up to the exciton bands. No vacuum ultraviolet 5d-4f emission of Nd was observed under excitation from 127 nm (10 eV) in BaF<sub>2</sub>, 120 nm (10.5 eV) in SrF<sub>2</sub> and 115 nm (11 eV) in CaF<sub>2</sub> down to 60 nm (20 eV) (see Fig. 1).

## B. Sm<sup>3+</sup> Emission

A weak luminescence of Sm<sup>3+</sup> ions was found in all the three hosts (Fig. 2) at both low and room temperature also. Two bands at 170–175 and 190–195 nm were observed. The first band shows a weak structure which is most prominent in the BaF<sub>2</sub>-Sm crystal. It can be assigned to the 5d-<sup>6</sup>H<sub>j</sub> transitions, while second band can be assigned to the 5d-<sup>6</sup>F<sub>i</sub> transitions.

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Fig. 3. Emission spectra of  $CaF_2$ ,  $SrF_2$ ,  $BaF_2$  crystals doped with 0.01 molar % of HoF<sub>3</sub> under 147 nm Xe-lamp excitation at 6.7 K. Sharp lines at 294 nm is due to second order of scattered exciting line at 147 nm.

The splitting of the  $5d - {}^{6}F_{j}$  transitions are twice less than that of  $5d - {}^{6}H_{j}$  transitions and correspondingly no structure was observed for it (see Fig. 2).

# C. $Ho^{3+}$ Emission

Emission spectra of  $\text{Ho}^{3+}$  show several bands below 200 nm and many sharp bands above 200 nm (see Fig. 3). The positions of the bands below 200 nm shift to the lower wavelength in a row from CaF<sub>2</sub> to BaF<sub>2</sub>, while the positions of the bands above 200 nm remain the same in all lattices. It is known that the outer 5d levels of rare-earth ions substantially influenced by surrounding lattice ions while the inner 4f levels only slightly depend on surroundings. Therefore one can undoubtedly conclude that the emission bands below 200 nm are due to the 5d-4f transitions while the bands above 200 nm belong to the 4f-4f transitions.



Fig. 4. Excitation and emission spectra of  $Tm^{3+}$  emission in  $CaF_2$ ,  $SrF_2$ ,  $BaF_2$  crystals. Higher intensity curves were measured with long time window (60–100 ns) while lower intensity curves—with short time window (2–10 ns). Inset show decay curves of Tm emission measured at 158 nm.

The 5d-4f emission bands was relatively weak in CaF<sub>2</sub> while they become more intensive in SrF<sub>2</sub> and BaF<sub>2</sub>. Besides the intensity of the 4f-4f emission bands decreases drastically from CaF<sub>2</sub> to SrF<sub>2</sub>, BaF<sub>2</sub> (see Fig. 3). Two Ho<sup>3+</sup> 5d-4f bands at 165 and 181 nm in BaF<sub>2</sub>, SrF<sub>2</sub> evidently belong to transitions to the <sup>5</sup>I<sub>8</sub> ground state and to the <sup>5</sup>I<sub>7</sub> first excited state of the 4f<sup>10</sup> configuration. Emission spectrum in CaF<sub>2</sub> is a combination of the 5d(LS)-4f and 5d(HS)-4f bands (see Fig. 3).

# D. $Tm^{3+}$ Emission

Time-resolved excitation and emission spectra (Fig. 4) show that a slow emission prevails in all bands. Only the 158 nm emission in CaF<sub>2</sub>-Tm shows a fast decay time (36 ns at 7K). No fast decay bands were observed in BaF<sub>2</sub> and SrF<sub>2</sub> (see Fig. 4). Therefore the 158 nm band belongs to an allowed transition from the 5d low-spin level, while the other intensive bands belong to a spin-forbidden transition from the 5d high-spin level.

The 5d-4f emission of Tm<sup>3+</sup> was excited only in the photon energy region below the exciton bands. No vacuum ultraviolet



Fig. 5. Emission intensity of  $SrF_2$  -NdF<sub>3</sub> crystals at maxima of exciton band (1) and Nd band (2) with x-ray excitation at room temperature. Inset shows emission spectra of  $SrF_2$  doped with 0.01 mol.% and 1 mol.% of NdF<sub>3</sub>.

5d-4f emission of  $\text{Tm}^{3+}$  was observed under excitation from 127 nm (10 eV) in BaF<sub>2</sub>, 120 nm (10.5 eV)in SrF<sub>2</sub> and 115 nm (11 eV) in CaF<sub>2</sub> down to 60 nm (20 eV) (see Fig. 4).

## E. Scintillation Efficiency

Emission spectra of Nd and Er doped alkaline-earth fluoride crystals under x-ray excitation were studied with concentration of impurity from 0.01 to 10 molar %. The integrated intensity of rare-earth ion emission was compared with that of the exciton emission, which is known to reach 10000 ph/MeV [11]. The most intensive emission of Nd<sup>3+</sup> and Er<sup>3+</sup> was observed in the SrF<sub>2</sub> crystals (Fig. 5). The Nd<sup>3+</sup> emission near 190 nm in BaF<sub>2</sub> is well overlapped with more intensive 196 nm band of crossluminescence and could not be correctly determined. For the Nd<sup>3+</sup> and Er<sup>3+</sup> 5d-4f emissions in SrF<sub>2</sub> the light yields were estimated as 30–40 ph/Mev. Light outputs for other rare earth ions was lower. These value are very small for the scintillator applications.

## IV. DISCUSSION

The zero-phonon lines of the Ho<sup>3+</sup> 4f-5d absorption and excitation lines were observed first in CaF2 at 157.1 nm [18] and 157.5 nm [19], respectively. No spin-forbidden transitions were observed in the excitation spectrum, possibly due to the low  $Ho^{3+}$  concentration [19]. To find the spin-forbidden bands we have measured excitation spectra for the integrated emission of all f-f bands within the 220-800 nm wavelength range of all three hosts with concentration of HoF<sub>3</sub> from 0.01 to 0.3 molar %. The spectra were measured at room temperature because the f-f emission under f-d excitation substantially decreases with decreasing temperature. New excitation bands were found on the long wavelength side of the first  $4f^{10}$ - $5d^14f^9$  bands with separation near 9 nm in all the hosts studied. The bands were weak in the samples with 0.01% of HoF3, but were evident in the samples containing 0.3% of HoF<sub>3</sub>. Evidently these excitation bands belong to the transitions to the high-spin  $5d^{1}4f^{9}$  states. The energy gaps between the Ho<sup>3+</sup>  $5d^{1}4f^{9}(HS)$  and  $5d^{1}4f^{9}(LS)$  states is near 3500 cm<sup>-1</sup> for all CaF<sub>2</sub>, SrF<sub>2</sub>, BaF<sub>2</sub> hosts, which is close to that observed in LiYF<sub>4</sub>-Ho (3460 cm<sup>-1</sup>) [20]. We conclude that the emission band near 158 nm in CaF<sub>2</sub>-Ho belongs to the spin-allowed transitions from  $5d^{1}4f^{9}(LS)$  and the most intensive emission band near 169 nm belongs to the spin-forbidden transitions from  $5d^{1}4f^{9}(HS)$  to the ground state (see Fig. 3). Energy splittings between the  $5d^{1}4f^{9}(HS)$  and  $5d^{1}4f^{9}(LS)$  emission and excitation bands in CaF<sub>2</sub>-Ho are the same. No spin-allowed transitions were observed in the SrF<sub>2</sub>-Ho or BaF<sub>2</sub>-Ho crystals (see Fig. 3).

In previous paper several emission bands in 150–220 nm region of alkaline-earth fluoride doped with Tm were observed and assigned to transition from  $4f^{11}5d^1$  to  $4f^{12}({}^{3}H_j, {}^{3}F_j)$  [12]. Weak emission near 160 nm were observed in CaF<sub>2</sub>-Tm [12]. In this paper we have measured the fast decay of this 160 nm emission band and slow decay of all other emission bands (see Fig. 4). It follows that the band near 160 nm belongs to allowed 5d-4f transitions from low spin 5d level, and the bands near 168 and 180 nm belong to spin forbidden transition from high spin 5d level.

The intensity of the 5d-4f vacuum ultraviolet emission of  $Nd^{3+}$  and  $Er^{3+}$  ions under x-ray excitation was maximal at concentration level 0.3-1%. However, this intensity becomes by one or two orders of magnitude smaller than the intensity of the exciton emission in undoped crystals. Therefore one can conclude that the mechanisms of energy transfer to 5d states of  $Nd^{3+}$ ,  $Tm^{3+}$  and  $Er^{3+}$  via excitons or electron-hole recombinations at impurity sites is not effective in alkaline-earth fluorides. Excitonic and consecutive electron-hole (or hole-electron) transfer mechanisms are the most efficient ones in scintillating crystals [14], [16]. In alkaline-earth fluorides, only the Ce ions could be excited by the exciton emission. For the Pr ions the electron-hole mechanism proves to be valid in alkaline-earth fluorides [17], [15]. However, the Nd and Tm 5d-4f emission excited by x-ray or band-to-band vacuum ultraviolet light was very weak (see Figs. 1, 4). It follows that the energy of electron-holes recombinations are enough for 4f-5d excitation of  $Pr^{3+}$  and not enough for excitation of  $Nd^{3+}$ ,  $Tm^{3+}$ .

## V. CONCLUSION

The 5d-4f emission of  $Ho^{3+}$  and  $Sm^{3+}$  was found in alkaline-earth fluorides. The main features of emission spectra can be satisfactorily explained by the transitions from the lowest energy d level to various terms of the 4f state. The highest-energy emission band at 158 nm in CaF<sub>2</sub>-Ho<sup>3+</sup> was assigned to the allowed transitions from the low-spin state. The transitions to the high-spin state was observed as weak excitation bands in all MeF<sub>2</sub>-Ho crystals.

The emission band near 160 nm with 36 ns decay time, observed in  $CaF_2$ -Tm, belongs to allowed transitions from the 5d low-spin level, while the other bands near 167 nm and 180 nm of alkaline-earth fluorides doped with Tm belong to a spin-forbidden transitions from the 5d high-spin level.

The absence of the excitation of the 5d-4f emission of Nd and Tm in  $CaF_2$ ,  $SrF_2$ ,  $BaF_2$  crystals as well as the weak light output of the 5d-4f scintillation of rare-earth ions indicate to the inefficiency of the electron-hole transfer mechanism.

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